



## Application of Multivariate Non-parametric Change-point Control Charts to Children with Bronchial Pneumonia

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### Abstract

Control charts are statistically and visually designed to detect changes or shifts in process. We apply the combined two nonparametric Rank Test of Wilcoxon-Mann-Whitney and Mood statistics called Lapage-type Change-Point (LCP) Chart. The modified chart shows great efficiency in detecting signals and shift in children's bronchial pneumonia, at about two years old (24th months), while it suggests that the actual shift had started at the 21st month (observation). This is an indication of the LCP promptness in raising alarm of a process shift if indeed it exists. The signal may have resulted due to shift in both mean ( $p=0.002949$ ) and variability ( $p=0.03978$ ) of children's bronchial pneumonia as measured. The study suggests that the new method should be used in short-run situations since it has the capacity of not only detecting shift but also the period it occurs and also where the underlying distributions are usually unknown.

**Keywords:** Wilcoxon-Mann-Whitney, Lapage-type Rank Test, Change-Point, Bronchial Pneumonia, force alarm

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### 1.0 Introduction

Quality control procedures are instituted to identify the areas and degree of imperfection, and thus assist in the interpretation of data, and may indicate the need for procedural changes (Chatterjee and Qiu, 2009). On the other hand, Statistical Process Control (SPC) is involved with continuous monitoring and or inspection of a process to ensure that the process mean or variance has not changed at any point in time (McCracken & Chakaraboti, 2012). Statistical Process Control (SPC) is about continuous monitoring or surveillance of a process to ensure that neither the mean nor the variability of the process distribution has changed (Hawkins and Zamba, 2005b) and (McCracken and Chakraborti, 2013).

Process control includes: Monitoring of some quality characteristics of the manufactured items to ensure compliance to certain standards; on-going surveillance of health data to detect an outbreak of a disease or increased rate of disease; the observance of a natural phenomenon such as changes in temperature. In public health practice, process control is very useful in showing variation that exists in health outcomes or performance between groups or institutions. Process control is often a starting point for needs assessment, for getting services and epidemiological understanding (Flower, 2009). Generally, the main goal of process control is to detect the changes in the process occurring at an unknown period of time as soon as possible after it has occurred



and simultaneously controlling the rate of false alarms (Dong et al. 2008).

Control charts are statistically and visually designed to detect changes or shifts in process. A process that is operating at or around some set values and only under some random variation (common causes) is said to be in an In-control (I.C) Process. On the other hand, process that somehow changes or shifts from in-control state is said to be out-of-control and usually denoted by OOC (Maravelakis, Panaretos & Psarakis, 2005). In SPC, it is essential to distinguish the variation due to real change of distribution (assignable or special cause) from that due to random error (chance or common cause). The procedure makes use of the control charts and this is necessary to establish whether there has been a significant change of any process distribution or not. Control charts, invariably are designed to serve this purpose by focusing on two strategies:

- (a.) To signal false alarm (false alarm occurs when a series signal while the process is in control),
- (b.) To signal as soon as possible when the process is Out-of-Control (OOC).

Having these two strategies in mind, the most popular technique for evaluating the performance of a control chart is the Average Run Length (ARL) which is based on the run length distribution (Maravelakis et al. 2005; Macracken & Chakraboti, 2013). Control charts as introduced by Walter Shewhart in 1924 are designed to detect changes or shift in a process through graphical display which allows a practitioner to determine whether a process is in control (IC) or out-of-control (OOC), this is done by taking samples at specified sampling interval and plotting the values of same statistics on a graphical interface which

includes decision and threshold times usually called control limits (Jensen, Jones-farmer, Champ, and Woodall, 2006). These charts are mostly designed to monitor individual variable say location (mean) or variability but not both; hence a concurrent use of two charts becomes certainly inevitable. In standard practice, this is done by pairs of control charts. For example, the Shewart  $\bar{X}$  charts is used to control the process mean and standard deviation chart (Park 2014) and (Reynold and Stoumbos, 2010). Using these paired charts (two charts) simultaneously provides a way to satisfy old-style of monitoring both the process means and variability at the same time. Despite the visible advantages of multivariate charts, single chart schemes have been established to be more attractive and preferable compared to the two-chart (combined charts) schemes due to the following reasons. Firstly, they are simple to apply by enabling practitioners to focus on single chart with a single variable, which makes the operation easier. Secondly, it is relatively easy to set the control limits for the chart based on the location (means) parameter and one for the variability (variance) parameter. Besides, they allow practitioners to avoid the inflated false alarm rate which results from simply using two independent control charts (McCracken & Chakaraboti 2013).

### **1.1 Non-Parametric Change-Point Approach**

When the assumptions from a parametric model are not met or known, nonparametric approach becomes more appropriate (Edokpa & Salisu, 2016). In practice, it is rare to expect that the assumptions will be exactly met or in fact that such information will be available to the practitioner. In this, section, a brief description of the existing non-parametric change-point formulation

considered is firstly given. Thereafter, the suggested control chart and its design are considered.

The change-point detection problem seeks to identify distributional changes at an unknown change-point  $k^*$  in a stream of data. The estimated change-point should be consistent with the hypothesis that the data are initially drawn from pre-change distribution  $P_0$  but from post-change distribution  $P_1$  starting at the change-point. This problem appears in many important practical settings, including bio-surveillance, fault detection, finance, signal detection, and security systems. (Rachel, Sara, Yajun, Rui, & Zhang, 2018)

### 1.2 Existing Non-Parametric-Based Change-Point Charts

SPC tools are conventionally used in one or two setting that is the Phase I. This is where we have data set of fixed size, of which the prior purpose is to estimate the in-control properties of the process reading. The Phase II involves a steady stream of incoming reading but conventionally does not involve any further refinement of the estimate of the in-control process behaviour. (Douglas, Hawkins & Qiqi, 2010).

To develop the non-parametric phase II methodology, it will be helpful to sketch the change-point formulation where you have a stationary data set, hence: assume  $X_1, X_2, X_3, \dots, X_r, X_{r+1}, X_n$  are independent continuous random variables with statistical distribution

$$X_i \sim F(x), \text{ for } i=1,2, \dots, r \quad 1.0$$

$$X_i \sim F(x, Q), \text{ for } i = r+1, 2, \dots, n \quad 1.1$$

The parameter  $Q$  represents a shift in location occurring after the change-point  $\gamma$ . But  $Q$  and  $\gamma$  are assumed unknown. Testing whether the process has shifted corresponds to the hypothesis test

$$H_0: Q = 0 \text{ Vs } H_1: Q \neq 0 \quad 1.2$$

or, equivalently, that the change-point  $\gamma$  lies outside the range  $1 \dots n$ .

### 1.3 Mann-Whitney (MW) Rank Test

Suppose that  $\{y_{11}, y_{21}, \dots, y_{n_1}\}$  and  $\{y_{21}, y_{22}, \dots, y_{n_2}\}$  are independent random samples from random variables  $Y_1$  and  $Y_2$  respectively. Let the combined sample of  $n = n_1 + n_2$  observations be  $Y = \{y_{11}, y_{21}, \dots, y_{n_1}, y_{12}, y_{22}, \dots, y_{n_2}\}$ .

Then, arrange and assign rank ( $R_i$ ) to the combined samples in ascending order of magnitude; where  $R_i$  is the rank of  $y_i$  (for  $i = 1, 2, \dots, n$ ). (Rotimi, Onoja & peter 2017)

Considering the distributions of the samples according to (Mukherjee and Chakraborti 2012) as:

$$F(y_1; \mu_1, \sigma^2) ; F(y_2; \mu_2, \sigma^2) \quad 1.3$$

Where  $\mu_1, \mu_2$  denote the location parameters,  $\sigma^2 > 0$  is the constant variability parameter and  $F(.)$  is a continuous distribution function. Under the null hypothesis that the two underlying populations have identical medians, the model (1.2) can be summarized in terms of a hypothesis as

$$H_0: \mu_1 = \mu_2 \text{ vs } H_1: \mu_1 \neq \mu_2 \quad 1.4$$

The Mann-Whitney rank test is thus expressed as:

$$M = \frac{U}{\sqrt{\frac{n_1 n_2 (n+1)}{12}}}$$

$$\text{where } U = \sum_{i=1}^{n_1} R_i - n_1(n+1)/2 \quad 1.5$$

The test statistic is in such a way that, depending on the null hypothesis, either a sufficiently small or a sufficiently large sum of ranks assigned to sample observations from population 1 causes  $H_0$  to be rejected.

### 1.4 Wilcoxon-Mann-Whitney test-Statistic

Suppose that  $\{Y_1, \dots, Y_k\}$  and  $\{Y_{k+1}, \dots, Y_t\}$  are independent random samples. We wish to test  $H_0: \mu_1 = \mu_2$  Vs  $H_1: \mu_1 \neq \mu_2$

Let  $R_1 < R_2 < \dots < R_k$  be the combined sample ranks of the first segment  $\{Y_1, \dots, Y_k\}$  observations in increasing order of magnitude. The Wilcoxon-Mann-Whitney rank test statistic for testing null hypothesis is defined as:

$$W = \sum_{i=1}^k R_i \quad 1.6$$

The mean and variance of the statistic  $W$  is given as:

$$E(W) = \frac{k(t+1)}{2} \text{ and } Var(W) = \frac{k(t+k)(t+1)}{12} \quad 1.7$$

The normalized statistic  $W$  is thus expressed as:

$$WM = \frac{\sum_{i=1}^k R_i - k(t+1)/2}{\sqrt{k(t-k)(t+1)/12}} \quad 1.8$$

The p-values for the Wilcoxon-Mann-Whitney test are based on the sampling distribution of the Run Sum statistic  $W$  when the null hypothesis (no difference in distributions) is true. Wilcoxon-Mann-Whitney is implemented in R function as “wilcox”. The process is thereafter considered to have shifted in location parameter given a lower p-value  $< \alpha$ , pre-specified level of significance.

### 1.5 Mood test Statistic

The mood statistic is used to test for a change in scale between two samples. Like the Wilcoxon-Mann-Whitney, the Mood test assesses the extent at which the ranks of the observations deviate from their expected value.

Suppose

that  $\{Y_1, \dots, Y_k\}$  and  $\{Y_{k+1}, \dots, Y_t\}$  are

independent random samples. We wish to test

$$H_0: \sigma_1 = \sigma_{11} \text{ Vs } H_1: \sigma_1 \neq \sigma_{11}$$

Let  $R_1 < R_2 < \dots < R_k$  be the combined sample ranks of the first

segment  $\{Y_1, \dots, Y_k\}$  observations in

increasing order of magnitude. Then the Mood test statistic for testing null hypothesis is defined as:

$$M = \sum_{i=1}^k \left( R_i - \frac{t+1}{2} \right)^2 \quad 1.9$$

The mean and variance of the statistic  $M$  is given as:

$$E(M) = \frac{k(t^2+1)}{12} \text{ and } Var(M) = \frac{k(t+k)(t+1)(t^2-4)}{180} \quad 2.0$$

The normalized statistic  $M$  is thus expressed as:

$$MD = \frac{M - k(t^2-1)/12}{\sqrt{\frac{k(t-k)(t+1)(t^2-4)}{180}}} \quad 2.1$$

The null distribution of  $M$  is needed to obtain the critical values and the p-value. Mood is implemented in R function as “mood test”. The process is thereafter considered to have shifted given a lower p-value  $< \alpha$ , pre-specified level of significance.

### 1.6 Wilcoxon-Mann-Whitney-Mood test Statistic

The concepts of Wilcoxon-Mann-Whitney depend largely on independent random sample and it is based on the sampling distribution of the rank sum statistic  $W$  when the null hypothesis (no difference in distributions) is true. The process is thereafter considered to have shift in location parameter given a lower p-value  $< \alpha$ , pre-specified level of significance. However, the mood statistic is used to test for a change in scale between two samples also; the Mood test assesses the extent at which the ranks of the observations deviate from their expected value. These are two nonparametric forms that can individually perform on the Mean and variance control charts.

Now let there be another form of model where the two forms are put together called

Lapage Rank test since both Wilcoxon-Mann-Whitney and Mood statistics are based on ranks and nonparametric form

### 1.7 Lepage-type Rank Test

The nonparametric two-sample Lepage test was developed by (Lepage 1971). The test is designed to carrying out equality of the location and scale parameters test simultaneously against the alternative that at least for one of the parameters, the equality does not hold. Basically, it is a combination of the Wilcoxon-Mann-Whitney and the Ansari-Bradley statistics (Hutchinson, 2002; Rublik, 2005). That is, it converts both the Wilcoxon-Mann-Whitney and the Ansari-Bradley statistics to square-standardized deviations from their respective expectations and adds the results.

Perhaps the most widely used two-sample rank test of equality of location parameters is the Wilcoxon-Mann-Whitney test. The Ansari-Bradley test is also used in two-sample rank test for equality of the scale parameters, though the ranking procedure is not that straightforward. However, the possibility of a two-sample test statistic which combines the Wilcoxon-Mann-Whitney and the mood statistics (with ranking procedure more direct and straightforward) has been suggested in the literature (Rublik, 2009). The nonparametric two-sample Lepage test's variant would be called Lepage-type test (for testing equality of the location and scale parameters against the alternative that at least for one of the parameters the equality does not hold) in the course of this study. Lepage-type test's proposition is straightforward in concept and simple to carry out.

Supposing that  $\{y_{11}, y_{21}, \dots, y_{n_1,1}\}$  and  $\{y_{21}, y_{22}, \dots, y_{n_2,2}\}$  are independent random samples from random variables  $Y_1$  and  $Y_2$

respectively. Assume their distributions are given as follows:

$$F(y_1; \mu_1, \sigma^2) ; F(y_2; \mu_2, \sigma^2)_{2.2}$$

where  $\mu_1, \mu_2$  denote the location parameters,  $\sigma_1^2 > 0, \sigma_2^2 > 0$  are scale parameters and  $F(\cdot)$  is a continuous distribution function. Let  $\{R_1, R_2, \dots, R_n\}$ ,  $n = n_1 + n_2$ , denote the rank of the pooled sample  $\{y_{11}, y_{21}, \dots, y_{n_1,1}, y_{12}, \dots, y_{n_2,2}\}$  random variables. Under the assumption of no change, this model (2.2) can be summarized in terms of a joint hypothesis as:

$$H_0: \mu_1 = \mu_2; \sigma_1 = \sigma_2_{2.3}$$

- a)  $H_1: \mu_1 \neq \mu_2 ; \sigma_1 = \sigma_2$  (representing shift in process location only)
- b)  $H_1: \mu_1 = \mu_2 ; \sigma_1 \neq \sigma_2$  (representing shift in process variability only)
- c)  $H_1: \mu_1 \neq \mu_2 ; \sigma_1 \neq \sigma_2$  (representing shift in both process location and variability)

### 1.8 Formulation of Lepage Rank Sum Type

Let  $T_w^2$  denote the square of the standard Wilcoxon-Mann-Whitney two-sample test statistic;  $T_M^2$  the square of the standardized mood two-sample test statistic; and L the combination of the Wilcoxon-Mann-Whitney and the Mood statistics, then Lepage-type test denoted by L is of the form:

$$L = T_w^2 + T_M^2_{2.4}$$

where

$$T_w^2 = \frac{(S_w - E(S_w / H_0))^2}{Var(S_w / H_0)}$$

$$T_M^2 = \frac{(S_M - E(S_M / H_0))^2}{Var(S_M / H_0)}_{2.5}$$

$$S_w = \sum_{i=1}^{n_1} R_i \text{ and } S_M = \sum_{i=1}^{n_1} \left( R_i - \frac{n+1}{2} \right)^2$$

$S_w$  and  $S_M$  are the Wilcoxon-Mann-Whitney and the Mood rank test statistic respectively; and  $E(\cdot)$  and  $Var(\cdot)$  denote the corresponding

expected value and variance of  $S_w$  and  $S_M$  under  $H_0$ . Thus;

$$E(S_w) = \frac{n_1(n+1)}{2} \text{ and}$$

$$Var(S_w) = \frac{n_1 n_2 (n+1)}{12}$$

$$E(S_M) = \frac{n_1(n^2+1)}{12} \text{ and}$$

$$Var(S_M) = \frac{n_1 n_2 (n+1)(n^2-4)}{180}$$

Hence, the Lapage-type statistic  $L$  of equation (2.4) could be expressed as:

$$L = \frac{12}{n_1 n_2 (n+1)} \left( S_w - \frac{n_1(n+1)}{2} \right)^2 + \frac{180}{n_1 n_2 (n+1)(n^2-4)} \left( S_M - \frac{n_1(n^2+1)}{12} \right)^2 \quad 2.6$$

## 2. The Proposed LCP Method

We suggest a nonparametric Lapage-type Change Point (LCP) approach to Statistical Process Control (SPC) based on Lapage-type test (which combines the Wilcoxon-Mann -Whitney and the Mood statistics for shift in process Mean and variability, respectively).

### 2.1 LCP Method Formulation

Suppose that the independent process observations  $\{y_1, y_2, \dots, y_n\}$  came from a continuous cumulative distribution function  $F(y; \mu_i, \sigma_i)$ , where  $\mu_i$  and  $\sigma_i$  are the Mean and variability parameters respectively. Also, consider the existence of time  $\tau$  (change-point) when there is a shift in mean and or in standard deviation of the process. The process reading, parallel Hawkins and Zamba (2005b), can be modelled as:

$$Y_i \sim \begin{cases} F(y; \mu_1, \sigma_1), & \text{if } i \leq \tau \\ N(y; \mu_2, \sigma_2), & \text{if } i > \tau \end{cases} \quad 2.7$$

Under the assumption of no shift, this model (2.3) can be summarized in terms of the hypothesis in equation (2.7) in which location shift occurs if  $\mu_1 \neq \mu_2$  and variability shift occurs if  $\sigma_1 \neq \sigma_2$ . If there is no enough evidence to reject the null hypothesis in equation (2.7), we will claim

that the process is in state of “statistical control” (in-control, IC) and stable with “random causes” which cannot be removed easily from the process without fundamental changes in the process itself. On the other hand, if it shows enough evidence to reject the null hypothesis in equation (2.7), we will conclude that the control chart issues a signal and the process is out of “statistical control” (out-of-control, OOC) and undergoes an unusual variation due to “special causes”. In principle, either or both of these shifts could occur. In addition to the  $\tau$  (change-point) being an unknown parameter,  $\mu_i$  (location parameter) and  $\sigma_i$  (variability parameter) are also unknown. Let the change-point  $\tau = k$  and  $R_i$  be the rank of  $y_i$  observations. And for the fact that the sample size keeps increasing in Phase II analysis as new observation comes in, we express the Lapage-type test statistic as:

$$L_{\max, n} = \max_k |L_{k, n}| \quad 2.8$$

where

$$L_{k, n} = \frac{12}{k(n-k)(n+1)} \left[ S_w - \frac{k(n+1)}{2} \right]^2 + \frac{180}{k(n-k)(n+1)(n^2-4)} \left[ S_M - \frac{k(n^2+1)}{12} \right]^2 \quad 2.9$$

$$S_w = \sum_{i=1}^k R_i \text{ and } S_M = \sum_{i=1}^k \left( R_i - \frac{n+1}{2} \right)^2$$

### 2.2 LCP Implementation Procedure

When the sample size is not fixed but increase, the procedure for adapting the formulation in equation (2.7) for use in the Statistical Process Control (SPC), setting is described similar to Hawkins and Zamba, (2005b) and (Ross, Tasoulis and Adams, 2011) as follows:

- (a) Find  $L_{\max, n}$ , after observation  $n$  has been added to the total record of the process, by

- i. Obtain the standardized

$$T_W = \frac{(S_W - k(n+1)/2)}{\sqrt{k(n-k(n+1))/12}}$$

and

$$T_M = \frac{(S_M - k(n^2 - 1)/12)}{\sqrt{\frac{k(n-k)(n+1)(n^2 - 4)}{180}}}$$

- ii. Calculate the sum of squares of the standardized statistics,  $L_{k,n}$
- iii. Determine the maximum of  $L_{k,n}$  over all the possible  $k$ ,  $L_{\max,n} = \max_k |L_{k,n}|$

- (b) If  $L_{\max,n} \leq h_n$ , where  $h_n$  is some suitable control limit, then conclude that there is no evidence of a shift in either mean or variance, and leave the process running uninterrupted.
- (c) If however,  $L_{\max,n} > h_n$ , then conclude that there is evidence of a shift in the mean, the variance or both.

One of the main objectives of a control chart is to detect unusual variation as soon as possible, and at the same time keeping the probability of erroneous signal below a reasonable level, using the initial framework of Hawkins, Qiu, and Kang (2003), while the process is in-control, the sequence of control limits ( $h_n$ ) is chosen so that the conditional probability of a false alarm at each observation  $n$  given that there was no false alarm prior to  $n$ , is fixed at desirably selected constant level  $\alpha$ . According to Dong, Hedayat and Sinha (2008), the type I error is usually characterized by the in-control average run length ( $ARL_o$ ) to a false

alarm. That is,  $ARL_o = \frac{1}{\alpha}$ . likening Hawkins and Zamba (2005b), this can be written in symbols as

$$P[L_{\max,n} > h_n | L_{\max,j} < h_j; j < n] = \alpha \quad 3.0$$

Theoretically, similar to the literature (such as Hawkins and Zamba, (2005a ,2005b); Zhou, Zou, Zhang and Wang (2009), Zamba and Hawkins (2006) submissions, it does not seem possible to solve for these  $h_n$  values. So, a simulation through the use of R “cpm” package Ross, (2013) is used to estimate them.

### 3. Application of the Methods to Children Suffering from Bronchial Pneumonia

There is no doubt that the challenges posed by bronchial pneumonia affect the entire population, but children are most vulnerable because of their unique physiologic, anatomic, morphologic and socio-economic characteristics (W.H.O, 2015). In the same vein, the Millennium Development Goals (MDGs) advance increased international attention focusing on child bronchial pneumonia in the developing world, with major aim to reduce under-five child mortality (Annim, Awusabo-Asare & Amo-Adjei, 2013). This assertion is characterized by the Nigeria situation particularly in the Savannah and Rain forest region of the country. The report by the World Health Organisation, 2016 resolved that National Development for Health Services (NDHS) in Hyper, Hypo and Meso-endemic area in Africa are charged with the responsibility of collecting data on the bronchial pneumonia of children. NDHS measure the weight of all children under age 5 (60 months) in selected households in Nigeria. The scope of the data used in the computation was based on the reported cases of bronchial pneumonia as obtained from the records section of Irrua Specialist Hospital.

Given the report and statistical submissions of the hospital in 2016, out of 30,050 children under age 5 in the 2016 routine health surveillance services on children about 17,345 were diagnosed with bronchial pneumonia representing 57.7% valid cases.

There are indications that the bronchial pneumonia status of children in Nigeria has gradually increased over the last decade. The extent of bronchial pneumonia has worsened informing recent decay in health care services. This submission is true as a result of climate change, humanitarian crisis, and socio-political instability being experienced in Nigeria. This is most common among children of the poor and average class accounting for 55-75% of the Nigeria population Centre for Disease Control (CDC) (2017). The proportion of children who suffered bronchial pneumonia increased from 11% in 2003 to 14% in 2008 and 18% in 2013 and the trend seems continuous particularly in the south-south region. Against this backdrop, the motivation to monitor the bronchial pneumonia status of children less than age 5 as measured by acute bronchial pneumonia using statistical quality control scheme becomes imperative.

#### 4.Data Application and Methods

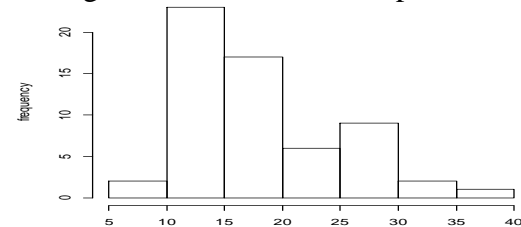
Centre for Disease Control (CDC) report in 2016 is adopted as spring board for change point control scheme in the study. The study considered the existing medical records on the report cases of children who suffered from the bronchial pneumonia disease in Irrua Specialist Hospital.

In addition, children with z-scores below minus two standard deviations from the reference population median are considered thin (bronchial pneumonia) or acutely pneumonia.

The performances of conventional change-point charts rely on the normality assumption of process distribution. However, sometimes, the distribution of process is not only skewed, but also heavy-tailed. In order to monitor bronchial pneumonia of age under-five children, its distribution is presented and assessed by a

histogram (see figure 1), below the most commonly used Shapiro-Wilk Normality Test.

**Figure 1.**A histogram of acute bronchial pneumonia of fewer than 5 children Histogram of acute bronchial pneumonia



Acute bronchial pneumonia (%)

#### Result

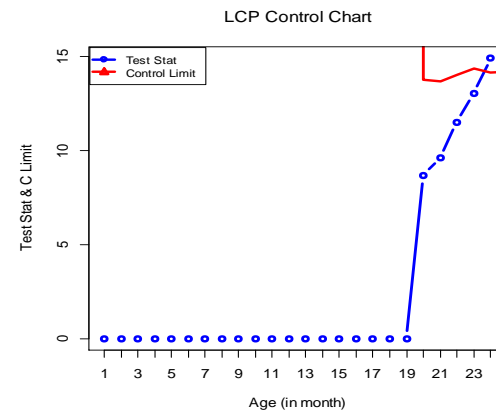
The Graph depicts that the data set distribution does not follow a normal distribution. Corroborating this, Shapiro-Wilk Normality test statistic:  $W= 0.9268$  ( $p=0.001463$ ) confirms that there is sufficient evidence to conclude that the data set has not been drawn from a normal population.

#### 4.1 Application of proposed LCP-based Chart

The figures 2a and 2b below respectively shows LCP nonparametric-based control chart applied to acute bronchial pneumonia of under 5 years children.

LCP nonparametric-based chart on acute bronchial pneumonia of under-5 years children

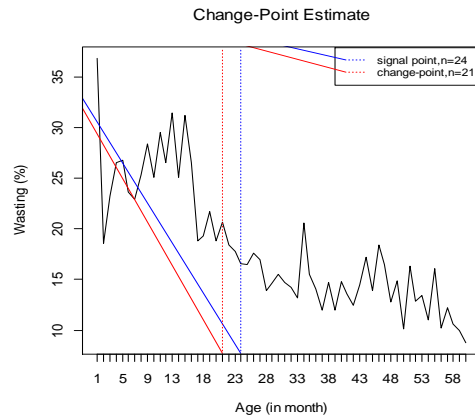
**Figure. 2a**





LCP estimated change-point, along with its detection time of acute of bronchial pneumonia of under-5years children.

**Figure. 2b**



**Results**

Figures 2a and 2b shows the results of applying nonparametric-based LCP control chart to the data. The chart does not only detect the shift in location but also the shift in variability. The estimated change-point, along with the period at which the maximised test statistic ( $L_{max}$ ) exceeds the control limit, is shown in figure b. The chart signals a shift in children’s bronchial pneumonia, as measured by acute pneumonia, at about two years old (24<sup>th</sup> month), while it suggests that the actual change had started at the 21<sup>st</sup> month (observation). This is an indication of the LCP promptness in raising alarm of a process shift if indeed it exists.

**After-Signal Diagnosis**

A shift signal by LCP is an indication of either shift in mean, in variance or in both. Hence, it is important for after-signal diagnosis to be carried out on the pre- and post-shift data segments.

Table 1 Summary Statistics of Pre-shift and Post-shift Segments of bronchial pneumonia

**Result**

The summary statistics in Table 1 confirm that the signal may have resulted in both a mean shift ( $p=0.002949$ ) and a variability shift ( $p=0.03978$ ) of children’s bronchial pneumoniaas measured.

**4.2 Discussion of Result**

	<b>Pre-Shift (Seg 1)</b>	<b>Post-Shift (Seg2)</b>	<b>2-sample (p-value)</b>
Mean	25.07201	17.33677	W = 76 (0.002949)
St.dev	4.747507	0.910598	Z = -2.056 (0.03978)

The inappropriateness of GLR control chart application to non-normal data is demonstrated with the use of Irrua Specialist Hospital data as applied to under-5 children bronchial pneumonia status, as measured by acute pneumonia diagnosis. The result is said to be unrealistic as the actual commencement of process monitoring was at 20<sup>th</sup> month. And indeed, this corroborated the stance of Hawkins and Zamba (2005b) claim that their proposed chart may not be suitable for non-normal data. Similarly, application of nonparametric-based MW chart to the data performed poorly. This could be attributed to the fact that there exist joint location and variability shifts in the process data. In agreement with literature such as Hawkins and Deng, (2009) and Zhang et al. (2010), MW could not in any way perform better compared with one-chart method designed to simultaneously monitor location and variability shifts in process quality. However, results from the LCP nonparametric-based chart substantiated NPC and ICT (2014) findings which reported that majority (83%) of children less than 6 months old were not exclusively monitored, and that over 90% of children



age 6-23 months were monitored inappropriately (based on recommended infant and young child monitoring practices). Bearing this in mind, application of LCP gave a clear pointer to the policy maker the need to urgently address the failure of children less than 2 years old to receive adequate monitoring. And, if the necessary corrective measures were taken to address this challenge, the tendency of wasting would be reduced to a large extent.

### 5. Conclusion

In conclusion, there is a clear indication that LCP performs quite better compared with the competing control charts. Traditional control charts have immensely contributed in the area of providing quality product and services, however, they lack the required ability to adequately and completely eradicate losses arising from let detection of faults due to false alarm and the period of such alarm.

### 6. Recommendation

The study suggests that the new method should be used in short-run situations, as in child bronchial pneumonia study, where the underlying distributions are usually unknown. We also recommend the method for both medical and manufacturers due to its high level of reliability since it has the ability of reducing the rate of false alarm.

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